On power distributions and stability of political mechanisms

Joseph Abdou

abdou@univ-paris1.fr

University of Paris 1- Paris School of Economics

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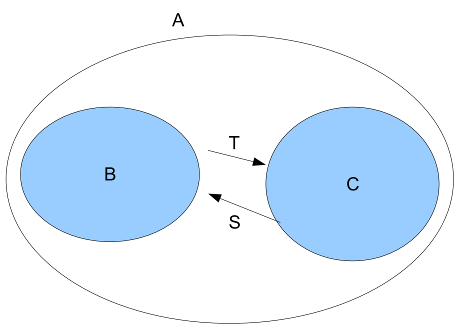
Motivation

- Stability is an essential requirement for political institutions; however it is both a theoretical and an empirical fact that political institutions are often unstable.
- Instability occurs when contradictory forces prevent the emergence of a persistent or a steady outcome, that is a state that is self-sustained and defendable when it is subject to attacks aiming to dismantle it.
- What can be observed when instability occurs is a volatile situation: any outcome that is proposed is subject to obstruction by some factions that block it. The result is a stalemate, where the institution is deadlocked.
- As far as governance or constitution of an established state is concerned, such a deadlock paves the way for a dramatic change of the institution itself; the latter is necessarily exogenous since no endogenous solution can be expected.
- However chaotic it may seem at first sight, instability in regulated systems presents some regularity and therefore there exist patterns of instability.

Patterns of instability: Bipolar deadlock



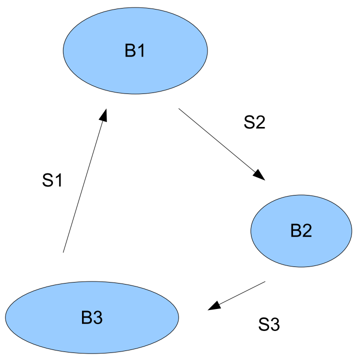
This is the pattern of a bipolar deadlock: Society is split into two groups. Each group can block a set of outcomes, but cannot force the complementary set, in such a way that all outcomes are blocked (Lebanon currently and under the Bush/Rice administration, Palestine since the rift between Gaza and the West Bank, Belgium and government formation a few years ago)

Game theoretic bipolar deadlock

	x_2	y_2
x_1	a	b
y_1	c	d

- Preferences of Player 1 are such that: $b, c \prec_1 a, d$
- Preferences of Player 2 are such that: $a, d \prec_2 b, c$
- The strategic game that results is a game with no pure Nash equilibria. The pattern of instability is bipolar. If b or c is proposed P1 would oppose them by threatening to achieve a or d. If a or d is proposed P2 would oppose them by threatening to achieve b or c.

Circular instability



This is the pattern of a circular configuration: S_1 opposes B_3 and prefers B_1, S_2 opposes B_1 and prefers B_2, S_3 opposes B_2 and prefers B_3.

Condorcet: typical tripartisan stalemate

- Majority voting among 3 alternatives leads to a “cycle” in collective choice in the sense that we can have

$$
\begin{aligned}
a &> b \\
b &> c \\
c &> a
\end{aligned}
$$

- where $x > y$ is a notation for “x is preferred to y by some majority ”

Strategic circularity

- The following game form is not strongly solvable:

	x_2	y_2
x_1	a	a
y_1	b	c

- For the following preferences it has no strong Nash equilibrium:
$$\begin{array}{ccccc} b & >_1 & a & >_1 & c \\ c & >_2 & b & >_2 & a \end{array}$$

- At any current state, the great coalition has the power to reach any other state. Thus a will be opposed by the great coalition that prefers b, b is opposed by P2, who prefers c and c is opposed by P1 who prefers a.

Towards a unifying model

- Interaction mechanisms

  1- Strategic game forms: Explicit strategies, mechanism, outcomes

  2 -Coalitional game forms = effectivity structures

  3- Interactive forms: unifying model that encompass essential features of both strategic and coalitional forms: No strategies, but description of power distribution over outcomes

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  2- Core for coalitional game forms
  3- Settlement set for general interaction forms

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- Interaction mechanism + Solution concepts $\longrightarrow$ Outcomes for every preference profile of the players

Part I: Notations

- $N = \{1, \ldots, n\}$ set of players (or agents)
- A is a finite set of alternatives (or social states)
- $\mathcal{P}_0(D) = \mathcal{P}(D) \backslash \{\emptyset\}$
- Q will denote some set of preorders on A
- If $R \in Q$, and $a, b \in A$ $a \overset{\circ}{R} b$ means that a is " strictly preferred" to b in the order R, $aR^{\sim}b$ means "a is equivalent to b"
- A preference profile is an element of Q^N.
- For every preference profile $R_N \in Q^N$ and $S \in \mathcal{P}_0(N)$ we put $Pr(a, S, R_N) = \{b \in A \mid b \ \overset{\circ}{R}{}^i \ a \ , \forall i \in S\}$

Example 1: Game forms and Equilibria

- We consider a strategic game form
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- A strategy array $x_N \in X_N$ is an $\mathcal{M}$-equilibrium of the game (G, R_N) if there is no coalition $S \in \mathcal{M}$ and $y_S \in X_S$ such that: $g(y_S, x_{S^c}) \in Pr(g(x_N), S, R_N)$.

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- Denote by $EO_{\mathcal{M}}(G, R_N)$ the set of $\mathcal{M}$-equilibrium outcomes.

Core(s) of a Game form

- An alternative a is in the $\mathcal{M}$-exact core of (G, R_N) if there is no coalition $S \in \mathcal{M}$ with the following property : for any $x_N \in X_N$ such that $g(x_N) = a$ there exists $y_S \in X_S$ such that $g(y_S, x_{S^c}) \in Pr(a, S, R_N)$.
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- An alternative a is in the $\mathcal{M}$-β-core of (G, R_N) if there is no coalition $S \in \mathcal{M}$ with the following property: for any $x_N \in X_N$, there exists $y_S \in X_S$ such that $g(y_S, x_{S^c}) \in Pr(a, S, R_N)$.
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  Denote by $C_{0,\mathcal{M}}(G, R_N)$, the $\mathcal{M}$-β-core of (G, R_N)
- In general:

$$EO_{\mathcal{M}}(G, R_N) \subset C_{1,\mathcal{M}}(G, R_N) \subset C_{0,\mathcal{M}}(G, R_N)$$

Why effectivity structures ?

In Example I, we are given (G, Sol), a couple where G is a game form and Sol a solution concept.

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Examples Sol = $\mathcal{M}$-equilibrium, $\mathcal{M}$-exact core or $\mathcal{M}$-beta core.

1. Determine the power distribution underlying (G, Sol): This power distribution is known as Effectivity structure
2. An effectivity structure describes the power distribution without direct reference to strategies.

(Local) Effectivity Function: definition

- A local effectivity function on (N, A) is a family $E \equiv (E[U], U \in \mathcal{P}_0(A))$ where for any $U \in \mathcal{P}_0(A)$, $E[U] : \mathcal{P}(N) \to \mathcal{P}(\mathcal{P}_0(A))$ and such that :
  (i) $E[U](\emptyset) = \{A\}$,
  (ii) $B \in E[U](S), B \subset B' \Rightarrow B' \in E[U](S)$,
  (iii) $U \subset V \Rightarrow E[V](S) \subset E[U](S)$.
- Let $R_N \in Q^N$. An alternative $a \in A$ is dominated at R_N if there exists $S \in \mathcal{P}_0(N)$,$U \ni a$ such that $Pr(a, S, R_N) \in E[U](S)$($a$ is objected to by S).
- The core of E at R_N is the set of undominated alternatives. It is denoted $C(E, R_N)$.

Local Effectivity of a game form

- Let G be a strategic game form. The local effectivity function $E^G_{1,\mathcal{M}}$ associated to $(G, \mathcal{M})$ is defined as follows: For $U \in \mathcal{P}_0(A)$,
- If $S \notin \mathcal{M}$: $E^G_{1,\mathcal{M}}[U](S) = \{A\}$,
- If $S \in \mathcal{M}$: $E^G_{1,\mathcal{M}}[U](S)$

$$= \{B \in \mathcal{P}_0(A) | \forall x_N \in g^{-1}(U), \exists y_S \in X_S : g(x_{S^c}, y_S) \in B\}$$

- The β-effectivity function associated to $(G, \mathcal{M})$ is defined by:

$E^G_{0,\mathcal{M}}(S) = E^G_{1,\mathcal{M}}[A](S)$ ($S \in \mathcal{P}(N)$).

Coincidence for core solutions

- The $\mathcal{M}$-exact core of (G, R_N) coincides with the core of $E^G_{1,\mathcal{M}}$ at R_N.
- The $\mathcal{M}$-β-core of (G, R_N) coincides with the core of $E^G_{0,\mathcal{M}}$ at R_N.

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- Effectivity structures represent well the underlying power in a game form when the solutions that are in force are of core types
- By contrast if the solutions that are in force are $\mathcal{M}$-equilibria then the effectivity structures are not adapted. This is why we introduce more precise interaction structures

A unifying model: Interaction Forms

- An interaction array on (N, A) is a mapping $\varphi : \mathcal{P}_0(N) \to \mathcal{P}(A)$.
- Let $\Phi = \Phi(N, A)$ be the set of all interaction arrays. We endow $\Phi(N, A)$ with the partial order $\leq$ where $\varphi \leq \varphi'$ if and only if $\varphi(S) \subset \varphi'(S)$ for all $S \in \mathcal{P}(N)$.
- The support of φ denoted $[\varphi]$ is the set of all $S \in \mathcal{P}_0(N)$ such that $\varphi(S) \neq \emptyset$.
- $\Phi_0(N, A) = \{\varphi \in \Phi(N, A) | [\varphi] \neq \emptyset\}$

Definition An interaction form over (N, A) is a mapping $\mathcal{E}$ from $\mathcal{P}_0(A)$ to subsets of $\Phi_0(N, A)$: $U \rightsquigarrow \mathcal{E}(U)$ satisfying the following conditions:
(i) $\varphi \leq \varphi', \varphi \in \mathcal{E}[U] \Rightarrow \varphi' \in \mathcal{E}[U]$

Interaction Form: interpretation

- Q-How is the proposition "$\varphi \in \mathcal{E}[\{a\}]$" to be interpreted ?
- A-To any state of the world that results in outcome a, there exists at least one coalition S that can move so that to send the outcome in $\varphi(S)$.
- Q- But does this mean that we can associate some $S = S(a)$, a coalition that achieves this action ?
- A- No! The information 'a is the current state" does not determine which coalition that achieves this action. Rather in some circumstances (undescribed here) S_1 could move, and in other circumstances S_2 could move, etc ... This is why we need all the vector or array φ

Solution for Interaction Forms

- An alternative a is dominated (opposed, dismantled, discarded ...) at the preference profile R_N if there exists some $U \ni a$, and some $\varphi \in \mathcal{E}(U)$ such that $\varphi(S) \subset P(a, S, R_N)$ for all $S \in \mathcal{P}_0(N)$.
- The alternative a is a settlement at R_N if it is not dominated at R_N. The set of all settlements at R_N will be denoted: $SET(\mathcal{E}, R_N)$.

From strategic GF to interactive form

- Given the strategic game form $G = (X_1, \ldots, X_n, A, g)$ and $\mathcal{M}$ an active coalition structure the associated interactive form $\mathcal{E}^G$ is defined as follows: For $U \in \mathcal{P}_0(A)$:

$$\mathcal{E}^G_{\mathcal{M}}[U] = \{\varphi \in \Phi(N, A) \,|\, \forall y_N \in g^{-1}(U), \exists S \in \mathcal{M}, \exists x_S \in X_S : g(x_S, y_{S^c}) \in \varphi(S)\}$$

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- **Lemma** (Coincidence of solutions) Let $G = (X_1, \ldots, X_n, A, g)$ be a game form. The set of $\mathcal{M}$-equilibrium outcomes of (G, R_N) coincides with the settlement set of $\mathcal{E}^G_{\mathcal{M}}$ at R_N.

From strategic GF to interactive form

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- **Lemma** (Coincidence of solutions) Let $G = (X_1, \ldots, X_n, A, g)$ be a game form. The set of $\mathcal{M}$-equilibrium outcomes of (G, R_N) coincides with the settlement set of $\mathcal{E}^G_{\mathcal{M}}$ at R_N.
- It is also possible to associate to any local affectivity function an interaction form that does the same job. Thus we have a unifying model.

Stability very abstractly

- We suppose that we are given the following objects:
  1- A finite set N (players, agents)
  2- A finite set A (alternatives, states)
  3- A (social choice) correspondence:
  $$H : Q(A)^N \to \mathcal{P}(A)$$
- For every $R_N \in Q(A)^N$, $H(R_N)$ is the (possibly empty) set of alternatives considered as the outcomes if the society has preference profile R_N.
- H represents the settlement set correspondence for some interaction form: $\mathrm{SET}(\mathcal{E}, \cdot)$
- For clarity, we are going to define the concepts of stability and stability index, abstractly on H.

Graded stability

- Let Π be the set of all partitions of A.
- If $\pi \in \Pi$ and $a \in A$ we denote by $\pi(a)$ the element of the partition that contains a.
- Define $Q_\bullet(\pi)$ to be the set of all $R \in Q(A)$ such that: $\pi(a) = \pi(b) \Rightarrow aR^{\sim}b$. The set of preferences compatible with the partition π .
- If $\pi \prec \pi'$ then $Q(\pi') \subset Q(\pi)$.
- For $\pi \in \Pi$, H is said to be π-stable if for any profile $R_N \in Q_\bullet(\pi)^N$, $H(R_N) \neq \emptyset$
- $r \in \mathbb{N}*$, H is said to be r-stable, if H is π stable for all partitions π of size r
- If H is r-unstable then H is $r+1$-unstable

Stability index: definition

Definition 1 *The stability index of H is the smallest integer r such that H is not r-stable. (By convention $= +\infty$ if H is r-stable for all $r \geq 1$)*

Graded stability for local eff functions

- In the case of local effectivity function take $H(\cdot) = C(E, \cdot)$. We can thus specify the foregoing definitions for this case:
- E is stable on $\mathcal{Q}$ if $C(E, R_N) \neq \emptyset$ for all $R_N \in \mathcal{Q}^N$.
- E is r-stable if $C(E, R_N) \neq \emptyset$ for all $R_N \in Q_\bullet(\pi)^N$ and all partitions π of size r.
- The stability index of E is the smallest integer r such that E is not r-stable ($+\infty$ if such an integer does not exist)
- The study of stability notions is easier for this model. Instability is closely related to the existence of some generalized Condorcet cycles

Cycles of a local effectivity function

- Let E ba a local effectivity function. An r-tuple $((U_1, B_1, S_1), \ldots, (U_r, B_r, S_r))$ where $r \geq 1$, $U_k \in \mathcal{P}_0(A)$, $B_k \in \mathcal{P}_0(A)$, $S_k \in \mathcal{P}_0(N)$ $(k = 1, \ldots, r)$ is a dominance configuration of E if:
  (i) $B_k \in E[U_k](S_k)$ $(k = 1, \ldots, r)$.
  (ii) $(U_1, \ldots, U_r)$ is cover of A.
- $(U_1, \ldots, U_r)$ is said to be the *basis* of the dominance configuration and r its length or order.
- A dominance configuration $((U_1, B_1, S_1), \ldots, (U_r, B_r, S_r))$ is a cycle of E if it satisfies the following property :
  (C) For any $\emptyset \neq J \subset \{1, \ldots, r\}$ such that $\cap_{k \in J} S_k \neq \emptyset$, there exists $k \in J$ such that for all $l \in J$: $B_k \cap K_l = \emptyset$.

Index of a local effectivity function

- **Theorem** The stability index of a local effectivity function E is equal to the minimal length of a cycle of E (with the convention that this number is $+\infty$ if E has no cycle)
- In particular E sis stable if and only if E is acyclic.

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(iv) $\sigma_0(G) = \sigma_1(G) = +\infty$ if and only if G is tight subadditive and exact.

- In particular $\sigma_0(G), \sigma_1(G) \in \{2, 3, +\infty\}$

Stability of $\mathcal{M}$-equilibrium: an outline

- Given a strategic game form G the local effectivity structure is not precise enough to reveal more than the exact core.
- In order to understand the underlying power in a strategic interaction under the rule of some $\mathcal{M}$-equilibrium we need the finer concept of power, namely the interaction form associated to the game form.
- Notions of cycles can be extended in a natural way to interaction forms.
- Cycles play the same role as in earlier results in describing stability and stability index
- details can be found in the Economic Theory paper

A flavor of the results

- $\sigma_S(G)$:= the stability index relative to the strong Nash equilibrium

Theorem 2 Let G be a game form. Then:

- (i) $\sigma_S(G) = 2$ if and only if G is not tight,
- (ii) $\sigma_S(G) = 3$ if G is tight and G is not subadditive,
- (iii) $\sigma_S(G) \leq r + 2$ if G is not r-exact $(1 \leq r \leq n)$.
- cf the Economic Theory paper

END

Selected related works

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