Observation Delays in Teams

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Introduction

General issue: collaboration in teams over time

- Two partners contribute to a project, which is commonly known to be good.
- The project ends for an agent if he observes a breakthrough, whose probability of occurrence depends on the level of effort exerted by the two partners at each date.
- Effort is costly, and not observable.

How do players allocate effort over time?

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Bonatti and Hörner (AER 2011): at the symmetric equilibrium,

- the equilibrium level of effort is constant over time;
- the equilibrium payoff is constant over time.

What we do

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Results

- There exists a unique symmetric equilibrium, in which players alternate between the maximal effort and the minimal effort.
- There exists a unique asymmetric equilibrium in which one player always exerts the maximal effort, and the other one never makes any effort.

Related literature

- Dynamic choice in teams:
  - Bonatti and Hörner: "Collaborating"(AER, 2011).
- Endogenous delay in teams:
  - Campbell, Ederer and Spinnewijn: "Delay and Deadlines: Free-riding and Information Revelation in Partnerships", (AEJ: Micro, 2014)
- Delay as a mechanism in teams:
  - Bimpikis and Drakopoulos: "Disclosing information in strategic experimentation", (2014).

The model

- Time is continuous, r common discount rate.
- Two agents $i = 1, 2$ continuously chooses a level of effort $k_t^i \in [0, \lambda]$, $\lambda \in]0, 1[$, over the infinite horizon $\mathbb{R}_+$.

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- A breakthrough is generated by agent i's effort at date t with instantaneous probability k_i^t.
- Effort k_i^t entails an instantaneous cost to agent i of αk_i^t with $\alpha > 0$.
- Effort decisions are not observable.

The model

If i observes a breakthrough at t then:

- he gets payoff 1 and the game stops for him.
- j will be informed of that only in $t + \Delta$ (if he is still playing then).
- If j has not observed a success before $t+\Delta$, then he gets a net present value 1 in $t + \Delta$ and the game stops for him.

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The probability that i observes a success at time t is:

$$\begin{aligned} &\text{if} \quad t < \Delta, \ e^{-\int_0^t k_s^i ds} k_t^i \\ &\text{if} \quad t \geq \Delta, \ e^{-\int_0^\Delta k_s^i ds - \int_\Delta^t k_s^i + k_{s-\Delta}^j ds} (k_t^i + k_{t-\Delta}^j) \end{aligned}$$

The model

Given a strategy profile (k^i, k^j), the discounted expected payoff to player i at time 0 is

$$U_0^i(k^i, k^j) = \int_0^{\Delta} e^{-rt} e^{-\int_0^t k_s^i ds} (k_t^i - k_t^i \alpha) dt$$

$$+ \int_{\Delta}^{\infty} e^{-rt} e^{-\int_0^{\Delta} k_s^i ds - \int_{\Delta}^{t} (k_s^i + k_{s-\Delta}^j) ds} (k_t^i + k_{t-\Delta}^j - \alpha k_t^i) dt$$

Best-response analysis

$$V_t^i = \max_{k_t^i}\{(k_t^i(1-\alpha)+\mathbb{1}_{t\geq\Delta}k_{t-\Delta}^j)dt+e^{-rdt}(1-(\mathbb{1}_{t\geq\Delta}k_{t-\Delta}^j+k_t^i)dt)V_{t+dt}^i\}$$

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After usual approximations

$$V_t^i = \max_{k_t^i}\{k_t^i[1-\alpha-V_{t+dt}^i]dt+\mathbb{1}_{t\geq\Delta}k_{t-\Delta}^j(1-V_{t+dt}^i)dt+(1-rdt)V_{t+dt}^i\}$$

- $k_t^i = \lambda$ is a best response if $V_{t+dt}^i \leq 1-\alpha$
- $k_t^i = 0$ is a best response if $V_{t+dt}^i \geq 1-\alpha$.
- The player is indifferent if $V_{t+dt}^i = 1-\alpha$.

NB: $1-\alpha$ is the equilibrium payoff without delay.

The case of impatient players

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Indeed,

$$U_0^i(\lambda, \lambda) = \frac{\lambda(1-\alpha)}{\lambda + r} + e^{-\Delta(\lambda+r)} \frac{\lambda(2-\alpha)}{2\lambda + r}$$

If $r \geq \frac{\alpha\lambda}{1-\alpha}$, then $U_0^i(\lambda, \lambda) \leq 1 - \alpha$ for all Δ, which implies that $k^i = \lambda$ is a best-response to $k^j = \lambda$. Since efforts are strategic substitute, $k^i = \lambda$ is a dominant strategy.

$\rightarrow$ in the rest of the paper, we consider relatively patient players who may postpone effort at some dates, namely

$$r \leq \frac{\alpha\lambda}{1-\alpha}$$

Preliminary result 1/2

Lemma There is no equilibrium in interior strategies: if (k^1, k^2) is an equilibrium, $k_t^i \in \{0, \lambda\}$ for all t.

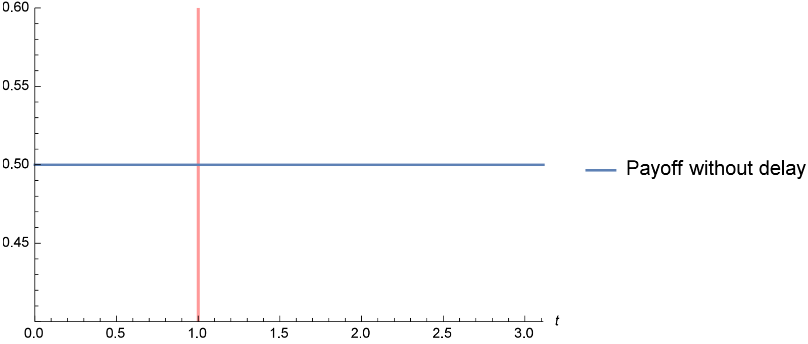
It follows that at any time t, player i is in one of these phases:

- an experimentation phase (E): $k_t^i = \lambda, k_{t-\Delta}^j = 0$;
- a waiting phase (W): $k_t^i = 0, k_{t-\Delta}^j = 0$;
- a free-riding phase (F): $k_t^i = 0, k_{t-\Delta}^j = \lambda$;
- a joint experimentation phase (J): $k_t^i = \lambda, k_{t-\Delta}^j = \lambda$.

Preliminary results 2/2

Lemma The payoff to player i is

- decreasing with time when i learns from j (in F and J phases).
- increasing with time when i does not learn from j (in E and W phases)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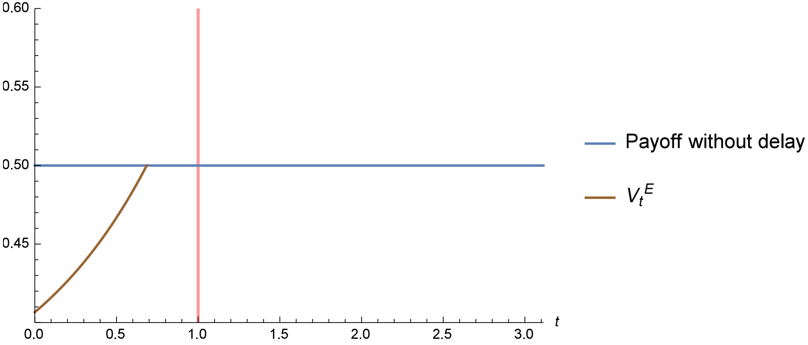
Payoff without delay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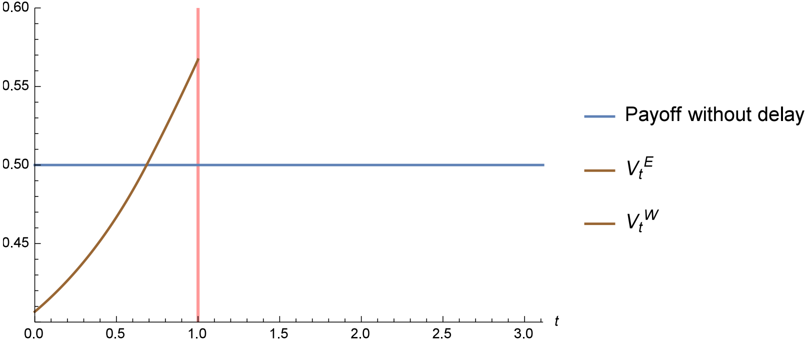
Payoff without delay
V_t^E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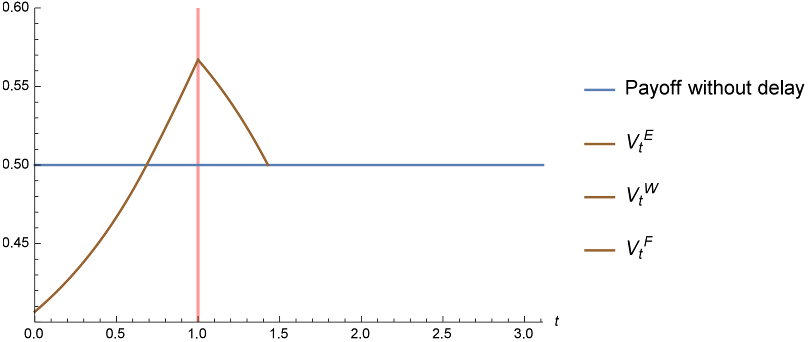

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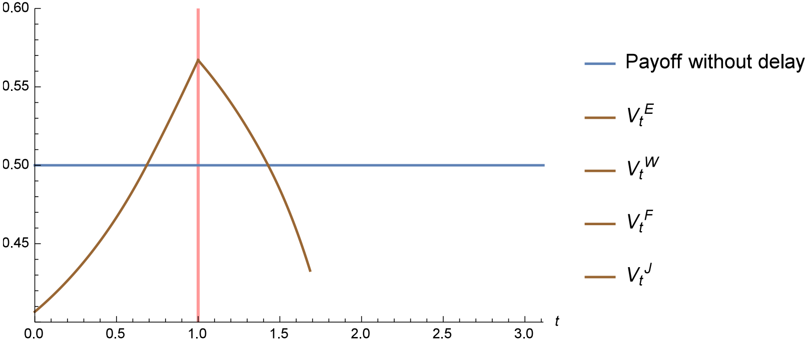
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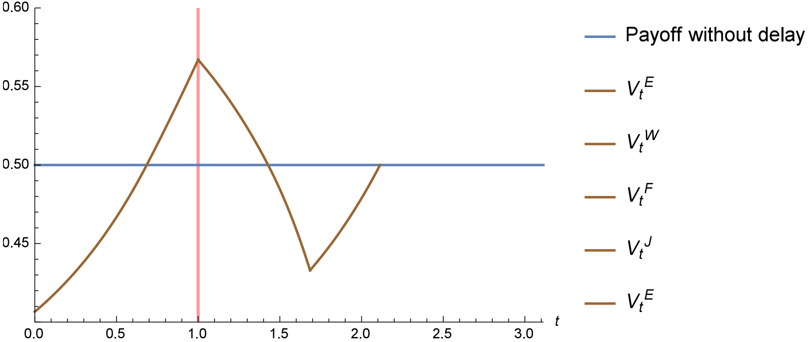
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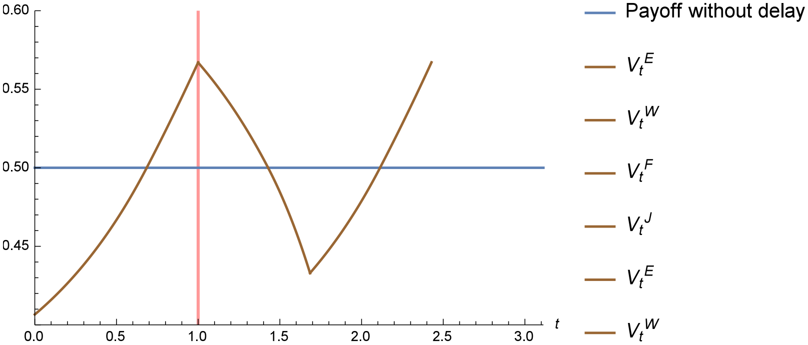
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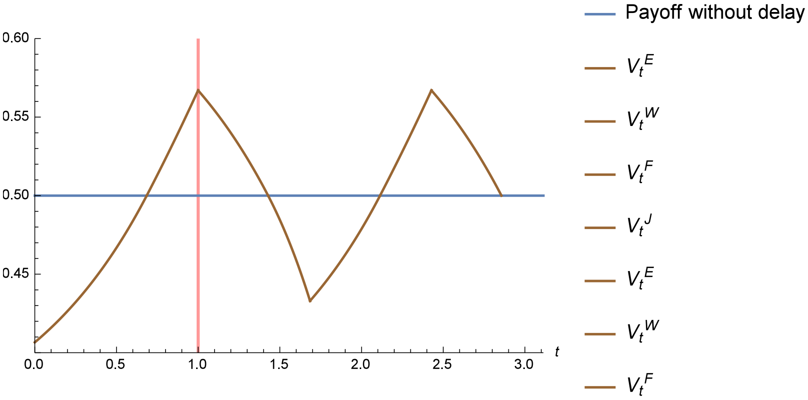
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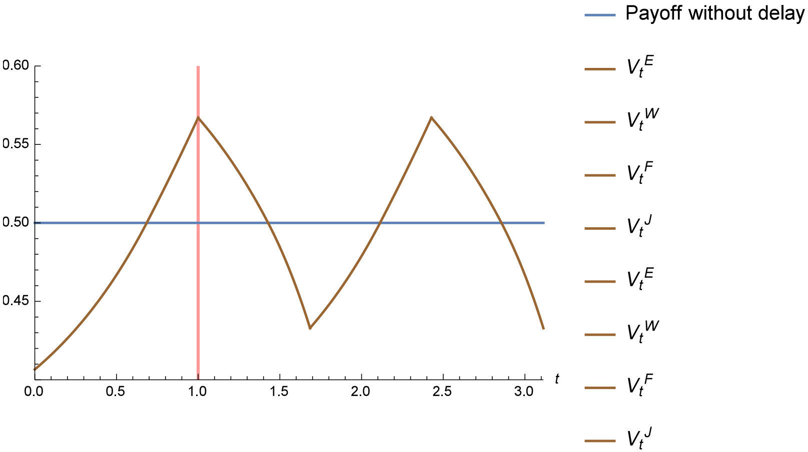

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Symmetric equilibrium 1/2

Theorem There exists a unique symmetric equilibrium.

The equilibrium strategy is cyclical and regular:

- players alternate effort phases $(k_t = \lambda)$ and rest phases $(k_t = 0)$.
- the length of effort phases and of rest phases is constant.

Symmetric equilibrium 2/2

Let ε and ρ denote the equilibrium values of effort and rest phases.

Lemma

- Delay increases the length of cycles: $\partial\varepsilon/\partial\Delta > 0$ and $\partial\rho/\partial\Delta > 0$.
- Delay hurts players payoff: $\partial U_0(k^*, k^*)/\partial\Delta < 0$
- Delay increases the amplitude of cycles: $\partial(\max_t V_t - \min_t V_t)/\partial\Delta > 0$
- Delay increases the effort intensity of cycles: $\partial\left(\frac{\varepsilon}{\varepsilon+\rho}\right)/\partial\Delta > 0$.

Asymmetric equilibrium

Theorem There is some bound $\bar{\Delta}$ such that

- If $\Delta \leq \bar{\Delta}$, there exists a unique asymmetric equilibrium in which one player plays $k^i = \lambda$ and the other one $k^j = 0$.
- If $\Delta > \bar{\Delta}$, there exists no asymmetric equilibrium.

Concluding remarks and open questions

Cycles also arise when:

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Cycles seem to come only from information delay.