# Invariant extensive-form rationalizability

Christina Pawlowitsch

Université Panthéon-Assas, Paris II,

joint with Rida Laraki

CNRS, Paris Dauphine et Ecole Polytechnique

French Symposium on Games , May 26–30, 2015

Pearce (1984): "Rationalizable strategic behavior and the problem of perfection"

– Rationalizability in the normal form (see also Bernheim, 1984)
– Rationalizability in the extensive form

Pearce (1984): "Rationalizable strategic behavior and the problem of perfection"

– Rationalizability in the normal form (see also Bernheim, 1984)
– Rationalizability in the extensive form

*"EFR has not received as much attention as normal-form rationalizability. There are at least two reasons for this neglect. One is simply that Pearce's definition of EFR is quite difficult to understand and tricky to handle in formal analysis. The other is that EFR has been seriously criticized for being inadequate in dealing with "counterfactuals" [..] Many games possess information sets that cannot be reached by rationalizable strategies, yet the players are assumed to optimize against "rational" conjectures also at these information sets"* (Battigalli, 1997).

## Our question:

- work with Pearce's original definition: simplify it and show some propoerties
- combine it with "invariance"

  $\rightarrow$ link this to "forward induction"

Definition (Pearce, 1984):

Let $H = (H^1, \ldots, H^n)$, $H^i \subset M^i$ closed, nonempty, and has the pure strategy property. For all $i = 1, \ldots, n$, define $H^i(0) = H^i$, and for any $t \geq 1$, define the sets $H^i$ recursively as follows: A strategy $m^i \in H^i(t-1)$ giving positive weight to pure strategies $s^i_1, \ldots, s^i_h$ is an element of $H^i(t)$ if for all $s^i_z$, $z = 1, \ldots, h$, for all $j \in J^i[s^i_z, H(t-1)]$, there exist conjectures $c^{ij}_z$, $c^{ij}_z \in \Pi^n_{r=1} \overline{H^r(t-1)}$, such that:

(C1) $c^{ij}_z(i) = s^i_z$ ($i$'s agent $j$'s conjecture about $i$'s own strategy is correct)

(C2) if $j \in J^i[s^i_z, H(t-1)]$ and $j \in J^i[s^i_{z'}, H(t-1)]$, then $c^{ij}_z(l) = c^{ij}_{z'}(l)$, $l \neq i$ (conjectures about other players' strategies do not depend upon which of the pure strategies $s^i_1, \ldots, s^i_h$ in the support of $m^i$ player $i$ ends up using),

(C3) for $j$, $j' \in J^i[s^i_z, H(t-1)]$, if $I^{ij}$ is a predecessor of $I^{ij'}$ and $c^{ij}_z$ reaches $I^{ij'}$, then $c^{ij'}_z = c^{ij}_z$,

(R1) $c^{ij}_z$ reaches $I^{ij}$, and

(R2) $s^i_z$ is a best response to $c^{ij}_z$ among all $ij$-replacement strategies for $s^i_z$ in $H^i(t-1)$.

Definition (Pearce, 1984):

Let $H = (H^1, \ldots, H^n)$, $H^i \subset M^i$ closed, nonempty, and has the pure strategy property. For all $i = 1, \ldots, n$, define $H^i(0) = H^i$, and for any $t \geq 1$, define the sets $H^i$ recursively as follows: A strategy $m^i \in H^i(t-1)$ giving positive weight to pure strategies $s^i_1, \ldots, s^i_h$ is an element of $H^i(t)$ if for all $s^i_z$, $z = 1, \ldots, h$, for all $j \in J^i[s^i_z, H(t-1)]$, there exist conjectures $c^{ij}_z$, $c^{ij}_z \in \Pi^n_{r=1}\overline{H^r(t-1)}$, such that:

(C1) $c^{ij}_z(i) = s^i_z$ ($i$'s agent $j$'s conjecture about $i$'s own strategy is correct)

(C2) if $j \in J^i[s^i_z, H(t-1)]$ and $j \in J^i[s^i_{z'}, H(t-1)]$, then $c^{ij}_z(l) = c^{ij}_{z'}(l)$, $l \neq i$ (conjectures about other players' strategies do not depend upon which of the pure strategies $s^i_1, \ldots, s^i_h$ in the support of $m^i$ player $i$ ends up using),

(C3) for $j$, $j' \in J^i[s^i_z, H(t-1)]$, if $I^{ij}$ is a predecessor of $I^{ij'}$ and $c^{ij}_z$ reaches $I^{ij'}$, then $c^{ij'}_z = c^{ij}_z$,

(R1) $c^{ij}_z$ reaches $I^{ij}$, and

(R2) $s^i_z$ is a best response to $c^{ij}_z$ among all $ij$-replacement strategies for $s^i_z$ in $H^i(t-1)$.

For each $i$, define

$$R^i(H) = \bigcap_{t=1}^{\infty} H^i(t),$$

*set of rationalizable strategies of player* $i$

## A hybrid process

.. a process that deletes strategies from the normal form of a game, but imposes extra conditions that come from the extensive form

.. decompose it into a process that operates on families of truncations of the normal form

# Example 1

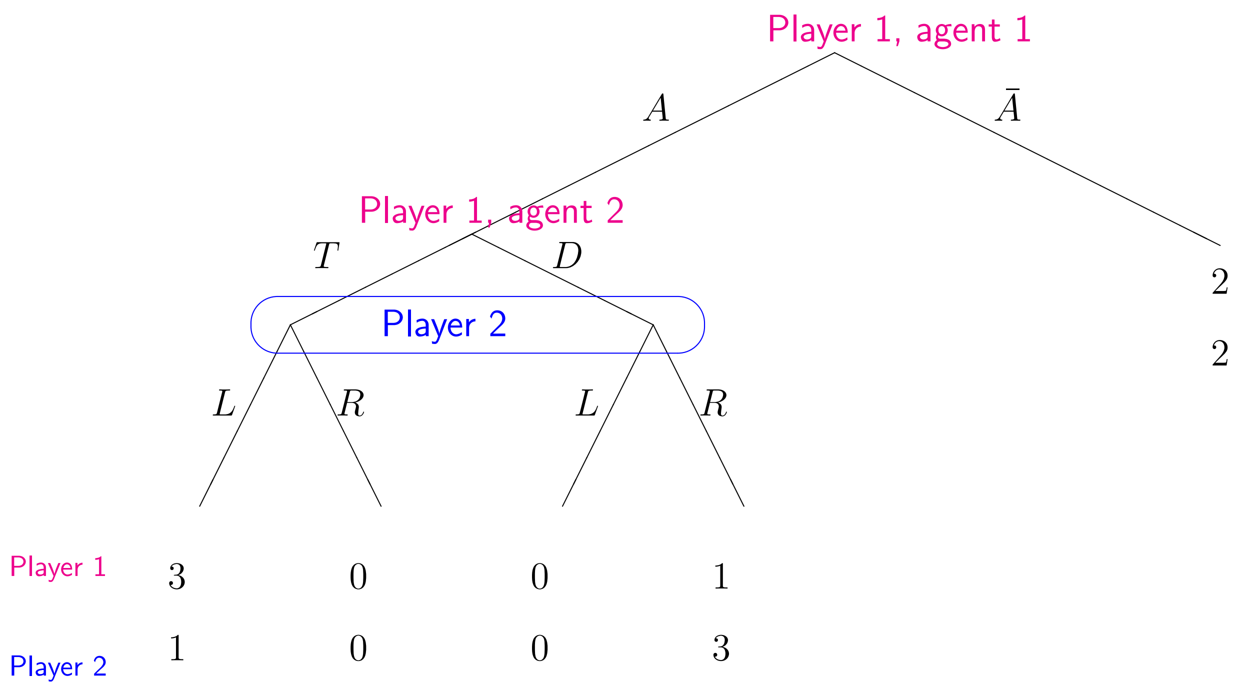


**Figure 1a. The Battle of the Sexes augmented by an outside option for player 1.**

| | L | R |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |
| $\bar{A}$ | 2,2 | 2,2 |

**Figure 1b. The normal form of the game in Figure 2a.**

## Normal-form truncations, step 1:

### associated to strategy $\bar{A}$:

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |
| $\bar{\mathbf{A}}$ | 2,2 | **2**,2 |

### associated to strategy $AT$:

| | $L$ | $R$ |
|---|---|---|
| **AT** | **3**,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |
| $\bar{A}$ | 2,2 | 2,2 |

| | $L$ | $R$ |
|---|---|---|
| **AT** | **3**,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |

### associated to strategy $AD$:

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| **AD** | 0,0 | 1,3 |
| $\bar{A}$ | 2,2 | 2,2 |

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| **AD** | 0,0 | **1**,3 |

### associated to strategy $L$:

| | **L** | $R$ |
|---|---|---|
| $AT$ | 3, **1** | 0,0 |
| $AD$ | 0,0 | 1,3 |

### associated to strategy $R$:

| | $L$ | **R** |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| $AD$ | 0,0 | 1,**3** |

## Normal-form truncations, step 2:

### associated to strategy $\bar{A}$:

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |
| $\bar{\mathbf{A}}$ | 2,2 | **2**,2 |

### associated to strategy $AT$:

| | $L$ | $R$ |
|---|---|---|
| **AT** | **3**,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |
| $\bar{A}$ | 2,2 | 2,2 |

| | $L$ | $R$ |
|---|---|---|
| **AT** | **3**,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |

### associated to strategy $AD$:

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| **AD** | 0,0 | 1,3 |
| $\bar{A}$ | 2,2 | 2,2 |

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| **AD** | 0,0 | 1,3 |

### associated to strategy $L$:

| | **L** | $R$ |
|---|---|---|
| $AT$ | 3, **1** | 0,0 |
| $AD$ | 0,0 | 1,3 |

### associated to strategy $R$:

| | $L$ | **R** |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| $AD$ | 0,0 | 1,**3** |

## Normal-form truncations, step 2:

### associated to strategy $\bar{A}$:

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |
| $\bar{\mathbf{A}}$ | 2,2 | **2**,2 |

### associated to strategy $AT$:

| | $L$ | $R$ |
|---|---|---|
| **AT** | **3**,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |
| $\bar{A}$ | 2,2 | 2,2 |

| | $L$ | $R$ |
|---|---|---|
| **AT** | **3**,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |

### associated to strategy $AD$:

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| **AD** | 0,0 | 1,3 |
| $\bar{A}$ | 2,2 | 2,2 |

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| **AD** | 0,0 | 1,3 |

### associated to strategy $L$:

| | **L** | $R$ |
|---|---|---|
| $AT$ | 3, **1** | 0,0 |
| $AD$ | 0,0 | 1,3 |

### associated to strategy $R$:

| | $L$ | **R** |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| $AD$ | 0,0 | 1,**3** |

## Normal-form truncations, step 3:

### associated to strategy $\bar{A}$:

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |
| $\bar{\mathbf{A}}$ | 2,2 | **2,2** |

### associated to strategy $AT$:

| | $L$ | $R$ |
|---|---|---|
| **AT** | **3**,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |
| $\bar{A}$ | 2,2 | 2,2 |

| | $L$ | $R$ |
|---|---|---|
| **AT** | **3**,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |

### associated to strategy $AD$:

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| **AD** | 0,0 | 1,3 |
| $\bar{A}$ | 2,2 | 2,2 |

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| **AD** | 0,0 | 1,3 |

### associated to strategy $L$:

| | **L** | $R$ |
|---|---|---|
| $AT$ | 3, **1** | 0,0 |
| $AD$ | 0,0 | 1,3 |

### associated to strategy $R$:

| | $L$ | **R** |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| $AD$ | 0,0 | 1,**3** |

## Normal-form truncations, solution:

**associated to strategy $\bar{A}$:**

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |
| $\bar{\mathbf{A}}$ | 2,2 | **2,2** |

**associated to strategy $AT$:**

| | $L$ | $R$ |
|---|---|---|
| **AT** | **3**,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |
| $\bar{A}$ | 2,2 | 2,2 |

| | $L$ | $R$ |
|---|---|---|
| **AT** | **3**,1 | 0,0 |
| $AD$ | 0,0 | 1,3 |

**associated to strategy $AD$:**

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| **AD** | 0,0 | 1,3 |
| $\bar{A}$ | 2,2 | 2,2 |

| | $L$ | $R$ |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| **AD** | 0,0 | 1,3 |

**associated to strategy $L$:**

| | **L** | $R$ |
|---|---|---|
| $AT$ | 3, **1** | 0,0 |
| $AD$ | 0,0 | 1,3 |

**associated to strategy $R$:**

| | $L$ | **R** |
|---|---|---|
| $AT$ | 3,1 | 0,0 |
| $AD$ | 0,0 | 1,**3** |

<table>
<tr><th>Step</th><th>Normal form at the beginning of the stage</th><th>Pearce's extensive-form rationalizability</th><th>"Forward Induction"</th></tr>
<tr><td>1</td><td>The original normal form:<br><table><tr><th></th><th>L</th><th>R</th></tr><tr><td>$AT$</td><td>3,1</td><td>0,0</td></tr><tr><td>$AD$</td><td>0,0</td><td>1,3</td></tr><tr><td>$\bar{A}$</td><td>2,2</td><td>2,2</td></tr></table></td><td>Delete $AD$: For player 1's agent 1, $AD$ has to be a best response to some conjecture in the general normal form. In this normal form, $AD$ is strictly dominated by $\bar{A}$, and hence, never a best response.</td><td>"player II knows that I will never choose $AD$, which is strictly dominated by $\bar{A}$."</td></tr>
<tr><td>2</td><td><table><tr><th></th><th>L</th><th>R</th></tr><tr><td>$AT$</td><td>3,1</td><td>0,0</td></tr><tr><td>$\bar{A}$</td><td>2,2</td><td>2,2</td></tr></table><br>. . . restricted to strategies of player 1 that reach player 2 (condition R1):<br><table><tr><th></th><th>L</th><th>R</th></tr><tr><td>$AT$</td><td>3,1</td><td>0,0</td></tr></table></td><td>Delete $R$: At this stage, for player 2, there exists no conjecture about player 1's strategy that reaches player 2 such that $R$ is a best response to that conjecture; obviously, for in the so defined truncation, $R$ is strictly dominated by $L$.</td><td>"if II sees he has to play, he should deduce that I [] certainly did not choose $AD$, where he was sure to get less than 2; player II should thus infer that I had in fact played $AT$ [..] and so player II should play L" [knowing that he cannot be playing $\bar{A}$, because this has been refuted by the observed move of I].</td></tr>
<tr><td>3</td><td><table><tr><th></th><th>L</th></tr><tr><td>$AT$</td><td>3,1</td></tr><tr><td>$\bar{A}$</td><td>2,2</td></tr></table></td><td>Delete $\bar{A}$: At this stage, there exists no conjecture for player 1's agent 1 such that $\bar{A}$ is a best response to that conjecture; in the remaining normal form, $\bar{A}$ is strictly dominated by $AT$.</td><td>"and hence player I should play AT."</td></tr>
</table>

<table>
<tr><th>Step</th><th>Normal form at the beginning of the stage</th><th>Pearce's extensive-form rationalizability</th><th>"Forward Induction"</th></tr>
<tr><td>1</td><td>The original normal form:
<table>
<tr><th></th><th>L</th><th>R</th></tr>
<tr><td>$AT$</td><td>3,1</td><td>0,0</td></tr>
<tr><td>$AD$</td><td>0,0</td><td>1,3</td></tr>
<tr><td>$\bar{A}$</td><td>2,2</td><td>2,2</td></tr>
</table></td><td>Delete $AD$: For player 1's agent 1, $AD$ has to be a best response to some conjecture in the general normal form. In this normal form, $AD$ is strictly dominated by $\bar{A}$, and hence, never a best response.</td><td>"player II knows that I will never choose $AD$, which is strictly dominated by $\bar{A}$."</td></tr>
<tr><td>2</td><td>
<table>
<tr><th></th><th>L</th><th>R</th></tr>
<tr><td>$AT$</td><td>3,1</td><td>0,0</td></tr>
<tr><td>$\bar{A}$</td><td>2,2</td><td>2,2</td></tr>
</table>
. . . restricted to strategies of player 1 that reach player 2 (condition R1):
<table>
<tr><th></th><th>L</th><th>R</th></tr>
<tr><td>$AT$</td><td>3,1</td><td>0,0</td></tr>
</table></td><td>Delete $R$: At this stage, for player 2, there exists no conjecture about player 1's strategy that reaches player 2 such that $R$ is a best response to that conjecture; obviously, for in the so defined truncation, $R$ is strictly dominated by $L$.</td><td>"if II sees he has to play, he should deduce that I [] certainly did not choose $AD$, where he was sure to get less than 2; player II should thus infer that I had in fact played $AT$ [..] and so player II should play L" [knowing that he cannot be playing $\bar{A}$, because this has been refuted by the observed move of I].</td></tr>
<tr><td>3</td><td>
<table>
<tr><th></th><th>L</th></tr>
<tr><td>$AT$</td><td>3,1</td></tr>
<tr><td>$\bar{A}$</td><td>2,2</td></tr>
</table></td><td>Delete $\bar{A}$: At this stage, there exists no conjecture for player 1's agent 1 such that $\bar{A}$ is a best response to that conjecture; in the remaining normal form, $\bar{A}$ is strictly dominated by $AT$.</td><td>"and hence player I should play AT."</td></tr>
</table>

## Example 2

"Burning Money" (Ben-Porath and Dekel, 1992). R1 and R2 come into play.

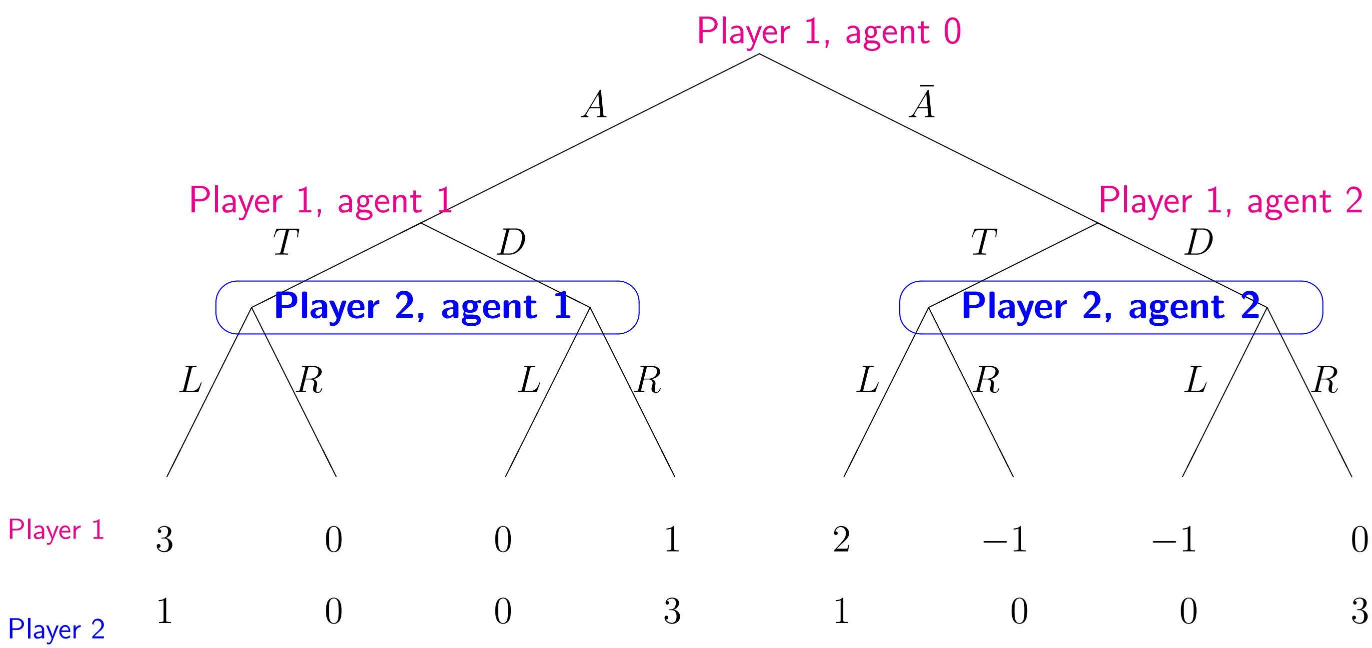


**Figure 2a. "Burning Money."**

| | LL | LR | RL | RR |
|---|---|---|---|---|
| $AT$ | 3,1 | 3,1 | 0,0 | 0,0 |
| $AD$ | 0,0 | 0,0 | 1,3 | 1,3 |
| $\bar{A}T$ | 2,1 | -1,0 | 2,1 | -1,0 |
| $\bar{A}D$ | -1,0 | 0,3 | -1,0 | 0,3 |

**Figure 2b. The normal form of the game in Figure 2a.**

<table>
<tr><th></th><th>Normal form at the beginning of the stage</th><th>Extensive-form rationalizability</th></tr>
<tr><td>Step 1</td><td>
<table>
<tr><th></th><th>LL</th><th>LR</th><th>RL</th><th>RR</th></tr>
<tr><td>$AT$</td><td>3,1</td><td>3,1</td><td>0,0</td><td>0,0</td></tr>
<tr><td>$AD$</td><td>0,0</td><td>0,0</td><td>1,3</td><td>1,3</td></tr>
<tr><td>$\bar{A}T$</td><td>2,1</td><td>-1,0</td><td>2,1</td><td>-1,0</td></tr>
<tr><td>$\bar{A}D$</td><td>-1,0</td><td>0,3</td><td>-1,0</td><td>0,3</td></tr>
</table>
</td><td>Delete $\bar{A}D$: For player 1's agent 0, $\bar{A}D$ has to be tested in the general, "big" normal form. In this normal form, $\bar{A}D$ is strictly dominated by a mixture of $AT$ and $AD$, and hence can never be a best response.</td></tr>
<tr><td>Step 2</td><td>
<table>
<tr><th></th><th>LL</th><th>LR</th><th>RL</th><th>RR</th></tr>
<tr><td>$AT$</td><td>3,1</td><td>3,1</td><td>0,0</td><td>0,0</td></tr>
<tr><td>$AD$</td><td>0,0</td><td>0,0</td><td>1,3</td><td>1,3</td></tr>
<tr><td>$\bar{A}T$</td><td>2,1</td><td>-1,0</td><td>2,1</td><td>-1,0</td></tr>
</table>
<br>R1: restricted to strategies of 1 that reach 2's agent 2:<br>
<table>
<tr><th></th><th>LL</th><th>LR</th><th>RL</th><th>RR</th></tr>
<tr><td>$\bar{A}T$</td><td>2,1</td><td>-1,0</td><td>2,1</td><td>-1,0</td></tr>
</table>
<br>R2: (a) Restricted to LR's replacement strategies: 2's agent-2 normal-form truncation in which $LR$ has to be tested:<br>
<table>
<tr><th></th><th>LL</th><th>LR</th></tr>
<tr><td>$\bar{A}T$</td><td>2,1</td><td>-1,0</td></tr>
</table>
<br>R2: (b) Restricted to RR's replacement strategies: 2's agent-2 normal-form truncation in which RR has to be a tested:<br>
<table>
<tr><th></th><th>RL</th><th>RR</th></tr>
<tr><td>$\bar{A}T$</td><td>2,1</td><td>-1,0</td></tr>
</table>
</td><td>(a) Delete LR: in the so truncated game, LR is strictly dominated by LL.<br><br>(b) Delete RR: in the so truncated game, RR is strictly dominated by RL.</td></tr>
</table>

<table>
<tr><td></td><td>Normal form at the beginning of the stage</td><td>Extensive-form rationalizability</td></tr>
<tr><td>Step 1</td><td><table><tr><td></td><td>LL</td><td>LR</td><td>RL</td><td>RR</td></tr><tr><td>$AT$</td><td>3,1</td><td>3,1</td><td>0,0</td><td>0,0</td></tr><tr><td>$AD$</td><td>0,0</td><td>0,0</td><td>1,3</td><td>1,3</td></tr><tr><td>$\bar{A}T$</td><td>2,1</td><td>-1,0</td><td>2,1</td><td>-1,0</td></tr><tr><td>$\bar{A}D$</td><td>-1,0</td><td>0,3</td><td>-1,0</td><td>0,3</td></tr></table></td><td>Delete $\bar{A}D$: For player 1's agent 0, $\bar{A}D$ has to be tested in the general, "big" normal form. In this normal form, $\bar{A}D$ is strictly dominated by a mixture of $AT$ and $AD$, and hence can never be a best response.</td></tr>
<tr><td>Step 2</td><td><table><tr><td></td><td>LL</td><td>LR</td><td>RL</td><td>RR</td></tr><tr><td>$AT$</td><td>3,1</td><td>3,1</td><td>0,0</td><td>0,0</td></tr><tr><td>$AD$</td><td>0,0</td><td>0,0</td><td>1,3</td><td>1,3</td></tr><tr><td>$\bar{A}T$</td><td>2, 1</td><td>-1,0</td><td>2, 1</td><td>-1,0</td></tr></table><br>R1: restricted to strategies of 1 that reach 2's agent 2:<br><table><tr><td></td><td>LL</td><td>LR</td><td>RL</td><td>RR</td></tr><tr><td>$\bar{A}T$</td><td>2,1</td><td>-1,0</td><td>2,1</td><td>-1,0</td></tr></table><br>R2: (a) Restricted to LR's replacement strategies: 2's agent-2 normal-form truncation in which $LR$ has to be tested:<br><table><tr><td></td><td>LL</td><td>LR</td></tr><tr><td>$\bar{A}T$</td><td>2,1</td><td>-1,0</td></tr></table><br>R2: (b) Restricted to RR's replacement strategies: 2's agent-2 normal-form truncation in which RR has to be a tested:<br><table><tr><td></td><td>RL</td><td>RR</td></tr><tr><td>$\bar{A}T$</td><td>2,1</td><td>-1,0</td></tr></table></td><td>(a) Delete LR: in the so truncated game, LR is strictly dominated by LL.<br><br>(b) Delete RR: in the so truncated game, RR is strictly dominated by RL.</td></tr>
</table>

<table>
<tr>
<td>Step 3</td>
<td>
<table>
<tr><th></th><th>LL</th><th>RL</th></tr>
<tr><td>$AT$</td><td>3,1</td><td>0,0</td></tr>
<tr><td>$AD$</td><td>0,0</td><td>1,3</td></tr>
<tr><td>$\bar{A}T$</td><td>2,1</td><td>2,1</td></tr>
</table>
</td>
<td>Delete $AD$: For player 1's agent 0, $AD$ has to be tested in the general, "big," normal form. At this stage, for player 1's agent 0, there exists no conjecture about player 2's strategy, still alive at this stage, such that $AD$ is a best response to that conjecture; in what remains of the game, $AD$ is strictly dominated by $\bar{A}T$.</td>
</tr>
<tr>
<td>Step 4</td>
<td>
<table>
<tr><th></th><th>LL</th><th>RL</th></tr>
<tr><td>$AT$</td><td>3,1</td><td>0,0</td></tr>
<tr><td>$\bar{A}T$</td><td>2,1</td><td>2,1</td></tr>
</table>
Restricted to strategies of player 1 that reach player 2's agent 1, which gives player 2's agent-1 normal-form truncation in which $RL$ has to be tested at this step:
<table>
<tr><th></th><th>LL</th><th>RL</th></tr>
<tr><td>$AT$</td><td>3,1</td><td>0,0</td></tr>
</table>
</td>
<td>Delete $RL$: in the so truncated game, $RL$ is strictly dominated by $LL$.</td>
</tr>
<tr>
<td>Step 5</td>
<td>
<table>
<tr><th></th><th>LL</th></tr>
<tr><td>$AT$</td><td>3,1</td></tr>
<tr><td>$\bar{A}T$</td><td>2,1</td></tr>
</table>
</td>
<td>Delete $\bar{A}T$: at this stage, for player 1's agent 0, there exists no conjecture about player 2's strategy, such that $\bar{A}T$ is a best response to that conjecture; in what remains of the game, $\bar{A}T$ is strictly dominated by $AT$.</td>
</tr>
</table>

## Order of deletion

Pearce's definition imposes a particular order of deletion: delete all strategies at once that do not pass the test

Unreached agents: no strategies can be deleted by force of that agent

## Order of deletion

Pearce's definition imposes a particular order of deletion: delete all strategies at once that do not pass the test

Unreached agents: no strategies can be deleted by force of that agent

We define: "extensive-form best responses"

$\rightarrow$ "iterated deletion of strategies that are never extensive-form best responses"

$\rightarrow$ independent of the order of deleton – as far as outcomes are concerned

## Properties of the normal form

Proposition: Let $\mathscr{E}$ be a game in extensive form. If at step $t$ of the process defined in Definition 3, respectively 5, if player $i$'s strategy $\check{s}^i = (\check{s}^{-ij}, \check{s}^{+ij}) \in H^i(t-1)$ is never an extensive-form best response, then it will be never an extensive-form best response for at least one agent $I^{ij}$, which is to say that for all $m^{-i} \in H_+^{-i}(I^{ij}, H(t-1))$,

$$\pi_i((\check{s}^{-ij}, \check{s}^{+ij}), m^{-i}) < \pi_i((\check{s}^{-ij}, \bar{s}_i^{+ij}), m^{-i})$$
$$\text{for some} \quad (\check{s}^{-ij}, \bar{s}_i^{+ij}) \in R[(\check{s}^{-ij}, \check{s}^{+ij}), H^i(t)],$$

and then:

(1) If $(\hat{s}^{-ij}, \check{s}^{+ij})$ is a strategy in $C[(\check{s}^{-ij}, \check{s}^{+ij}), H^i(t-1)]$ that *does not* preclude $I^{ij}$ from being reached (which includes $(\check{s}^{-ij}, \check{s}^{+ij})$ itself), then:

(a) for all $m^{-i} \in H^{-i}_{+I^{ij}}(t-1)$:

if $(\hat{s}^{-ij}, s^{+ij}) \in R[(\hat{s}^{-ij}, \check{s}^{+ij}), H^i(t-1)]$ and
$(\bar{s}^{-ij}, s^{+ij}) \in R[(\bar{s}^{-ij}, \check{s}^{+ij}) H^i(t-1)], (\bar{s}^{-ij}, \check{s}^{+ij}) \in C[(\check{s}^{-ij}, \check{s}^{+ij}), H^i(t-1)]$
then:
$\pi_j[(\hat{s}^{-ij}, s^{+ij}), m^{-i}] = \pi_j[(\bar{s}^{-ij}, s^{+ij}), m^{-i}]$, for all players $j = 1, \ldots, n$;

(b) for all $m^{-i} \in H^{-i}_{-I^{ij}}(t-1)$:

$$\pi_j[(\hat{s}^{-ij}, \check{s}^{+ij}), m^{-i}] = \pi_j[(\hat{s}^{-ij}, s^{+ij}), m^{-i}] \quad \text{for all} \quad (\hat{s}^{-ij}, s^{+ij}) \in R[(\hat{s}^{-ij}, \check{s}^{+ij}), H^i(t-1)],$$
$$\text{for all players } j = 1, \ldots, n.$$

(3) For a strategy $(\hat{s}^{-ij}, \breve{s}^{+ij})$ in $C[(\breve{s}^{-ij}, \breve{s}^{+ij}), H^i(t-1)]$ that *precludes* $I^{ij}$ from being reached:

for all $m^{-i} \in H^{-i}(t-1)$:

$$\pi_j[(\hat{s}^{-ij}, \breve{s}^{+ij}), m^{-i}] = \pi_j[(\hat{s}^{-ij}, s^{+ij}), m^{-i}] \quad \text{for all} \quad (\hat{s}^{-ij}, s^{+ij}) \in R[\hat{s}^{-ij}, \breve{s}^{+ij}, H^i(t-1)],$$

for all players $j = 1, \ldots, n$.

## Invariance

Invariance of the semi-reduced normal form (Tompson transformations)

$\rightarrow$ apply the condition in the proposition as a necessary condition

$\rightarrow$ something non-empty

$\rightarrow$ depends on the order of deletion only as far as outcomes are concerned

$\rightarrow$ "invariant extensive-form rationalizable strategies"