Target Bidding and Collusion in the Vickrey Auction

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Outline

1. Motivation
2. Gap Rule
3. Auction with Target Bids
4. Summary

Vickrey Auction

- Vickrey (second-price) auction has many good properties
- But: very susceptible to collusion among bidders
- Collusion can drastically reduce the seller's revenue
- There exists no universal solution

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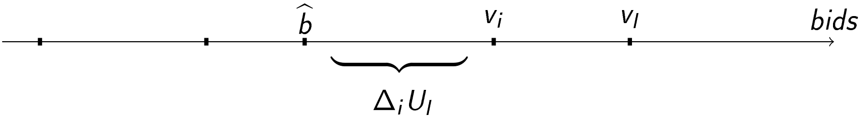

Figure : Collusion surplus when bidder i withdraws his bid v_i.

Gap Rule

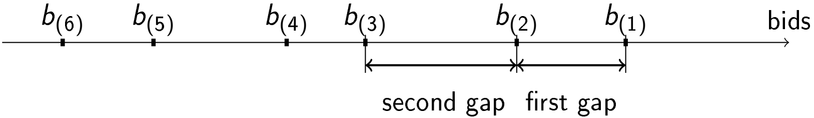

Figure : *Gap Rule.* $b_{(k)}$ *denotes* k^{th}*−highest bid. If the second gap is larger than the first gap, the second bidder wins at price* $b_{(3)}$.

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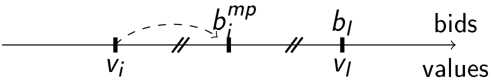
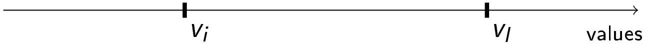


Figure : *Midpoint bid* $b_i^{mp} = \frac{v_i + v_1}{2}$

Gap Rule

Fix any valuations profile such that $v_i < v_l$ and let $\hat{b} = \max\{v_j\}_{j \in N/\{l,i\}}$, where N is the set of all bidders, $|N| \geq 3$. Under the gap rule, bidding $b_i^{mp} = \frac{v_i + v_l}{2}$ yields payoff $\Delta_i U_l$ to bidder i.

Gap Rule



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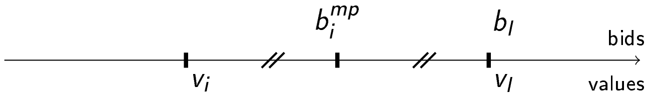

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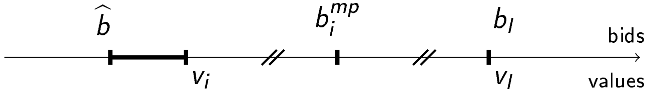

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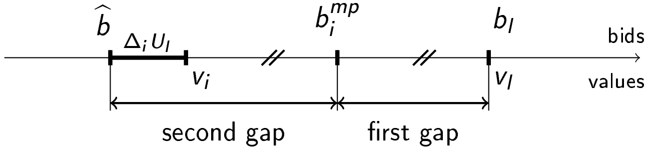
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values
second gap
first gap

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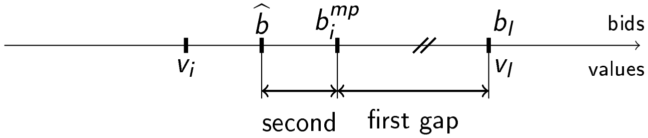

Gap Rule

Bidder i's payoff under GR coincides exactly with the collusion surplus in Vickrey

Problem!

This assignment rule is inefficient.

GR has to be built into an efficient auction and triggered whenever collusion is present

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Auction Rules: 2 Rounds

Round 1 Each bidder i submits a preliminary bid $\beta_i \geq 0$ and a target bidder identity $j \equiv T(i) \in N$ (self-targeting is permitted).

Recursively define $T^{k+1}(i) = T\left(T^k(i)\right)$. Bidder i's *target set* as $\mathbb{T}(i) = \cup_{k=1}^{n} T^k(i)$, and i's target bid: $\underline{\beta_i} = \min\{\beta_j, j \in \mathbb{T}(i)\}$.

Round 2 The final bid b_i of bidder i is determined as follows:

$$b_i = \min\left\{\frac{1}{2}\left(\beta_i + \underline{\beta_i}\right); \underline{\beta_i}\right\}$$

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Auction Rules: Assignment

- If two highest bidders self-target, then the Vickrey rule is implemented. Otherwise, the gap rule is implemented. If the resulting price is lower than the seller's cost, she keeps the object.
- If there is a tie at the top, and M is the set of tying bidders, the winner is picked at random from the intersection of target sets $\cap_{i \in M} \mathbb{T}(i)$ and the winner pays her bid. Otherwise, if there is a tie in the second bid, both rules assign the object to the highest bidder at the price of the second bid.

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Example 1

Suppose there are three bidders: B1, B2, and B3, who bid respectively 1, 2 and 3 in the first round and target their own bids. The Vickrey rule is applied and B3 wins at price 2.

Example 2

As in example 1, except that B2 targets B3. The final bids are $(1, 2\frac{1}{2}, 3)$. In this case, gap rule applies and B2 wins at price 1.

Example 3

As in example 1, except that B2 targets B1. The final bids are $(1, 1, 3)$. B3 wins at price 1.

Example 4

As in example 1, except that B3 targets B2. The final bids are $(1, 2, 2)$, there is a tie. $\mathbb{T}(B2) \cap \mathbb{T}(B3) = \{B2\}$, hence B2 wins at price 2.

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BNE: Everybody self-targets and bids their true valuations.

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Outcome-equivalent to the Vickrey auction (non-cooperative)

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Fix any cartel, a proper subset of the set of players. There exists no incentive feasible C-Pareto improving side contract.

- Intuition: Target bidding serves as a whistle-blowing device.
- The whistle-blower is rewarded by getting the opportunity to win the auction at an attractive price.
- The cartel leader cannot pay enough bribes to preclude deviations, unless he pays more than his collusion surplus.

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  - Coin flip
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Related Literature

1. Efficient organization of collusion: Graham, Marshall (1987), Mailath, Zemsky (1991), McAfee, McMillan (1992)
2. Bribes: Schummer (2000), Esö, Schummer (2004), Rachmilevitch (2013)
3. Strong notion of collusion robustness: Laffont, Martimort (1997, 2000), Pavlov (2008), Che, Kim (2009)
4. Information disclosure and collusion in auctions: Marshall, Marx (2007)
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